1. The phrase "calculated risk" ~~is a military~~ occurs in military writing--specifically, in military autobiographies--to describe decisions that led to horrible fiascos. For example, Truman's comment on looking down from the cliffs over Salerno last year was that the "squirrelheaded general" who decided to land on those beaches must have been insane. But the squirrelheaded general himself recalled that the "whole Allied campaign in Italy was a vast calculated risk"; in fact, he chose to entitle his autobiography "Calculated Risk."

2. It's common to sneer at such pretensions of calculation, as describing real decision-making under uncertainty. In the words of one writer:

> "I conceive the nature of expectation...as an act of creative imagination and not of 'rational' calculation; for calculation is impossible when the data are incomplete, and in face of ignorance, rationality is a mere pretense." Shackle

3. And Clark himself goes on to say that the assumptions that underlay the Allied plans were "mere wishful thinking"; which suggests that the appearance of calculation might have been deceptive.

4. Such skepticism, however, cuts just as deeply into economic theories of decision-making; for these theories have been based almost entirely on the notion of the calculated risk. In fact, the quotation above is by an economist, who was referring to economic behavior and criticizing economic theories.

5. Until very recently, neither the proponents ~~of th~~ nor the critics of this approach relied very heavily on any sort of empirical evidence, so they never bothered to spell out very closely just how one would go about testing or applying their theories. Just what are they asserting, in terms of observable evidence? Do the various approaches really conflict; i.e., do they lead to different predictions? If so, when; and which seems, at this point, more likely to be right? What sort of data are required, what are the operations for measuring them?

6. I am going to suggest that there are phenomena that lie outside the familiar models of the calculated risk, and that models exist that may be useful in describing them. But before I can stake out an area of aplication for the Theory of the Uncalculated Risk, I must indicate the empirical content of the Theory of the Calculated Risk, the specific implications of that approach that I expect to see violated.

~~7~~

7. First, what are the variables to be calculated? ~~On this there is very wide agreement.~~ Clark defines acalculated risk as ((a decision in which the advantages outweigh the disadvantages). There is very wide agreement that this leaves something out: a particular action may have many possible advantages and disadvantages, but in making his decision the person weights these various outcomes ~~in~~ on the basis of "likelihood," what Shackle calls,

> "The strength of their claim to be treated as if they were true."

8. These, then, are the materials that go into the calculation: the attractiveness of the possible outcomes, and some sort of judgments of their likelihoods. As Ramsey puts it, the theory assumes that:

> "We act in the way we think most likely to realize the objects of our desires, so that a person's actions are completely determined by his desires and opinions."

9. The next question~~si~~ is, how do we measure these variables, and how much do we need to know about them? Suppes and Davidson, in their recent book on decision-making, begin with the statement:

> "When an individual chooses between alternatives which involve uncertain outcomes it seems clear that at least two factors enter the decision: the degree to which the possible outcomes are desired relative to one another, and the degree to which the outcomes are deemed probable."

10. This emphasis on degree is common to nearly all discussion of the calculated: the notion of weighing increments against each other, of comparing intervals, nearly always crops up. In other words, some knowled that goes beyond an ordering of outcomes in terms of desirability and likelihood is involved. It is implied that some process of measurement has established an interval scale for these magnitudes, so that it is meaningful to say that A is preferred to B more than C is preferred to D; in fact, it is usually considered meaningful to say that event X is twice as probable as event Y, which implies a ratio scale for probability. It is usually assumed that the person can be described as though he made his decision by multiplying and adding certain numbers corresponding to the desirabilty and likelihood of the outcomes.

11. Now, where did these numbers come from? What are the operations

12. Probs based on statistical knowledge, desirability measured by introspective operation or in terms of money.

13. But Knight questioned both the existence ~~and~~ of such statistical knowledge and its relevance to individual decisions. He still seems to agree that decisions are ~~on~~ made on the basis of some notion of likely outcomes, but emphasizes that these "estimates of likelihood" vary from person to person and bear no close relation to statistical frequencies. He rejects numerical probs mainly because, having eliminated information on numerical frequencies, he sees no way to measure them. This leads him to distinguish between ~~risk~~ "measurable" risk and "unmeasurable" uncertainty.

14. Shackle carries this same argument even further, to the point of saying that in many situations--specifically, situations in which all the outcomes seem "possible," so that they wouldn't cause "surprise," the person behaves as if he didn't even have an ordering of likelihoods. In throwing a die, for instance, it wouldn't surprise us at all if, on the one hand, a one or a two came up, ~~and~~ nor if, on the other hand, a higher number came up. So Shackle concludes:

"Suppose the captains in a Test Match have agreed that instead of tossing a coin for choice of innings they will decide the matter by this next throw of a die, and that if it shows an ace Australia shall bat first, if any other number, then England shall bat first. Can we now give any meaningful answer whatever to the question "Who will bat first?' except 'We do not know?"

15. Another group pointed to situations where they said that numerical probs were not merely unknown or irrelevant, but inherently meaningless, undefinable. Von Neumann and Morgenstern took the position that, whereas statistical assumptions were perfectly fine in non-game situations, it was somehow impossible to introduce numerical probabilities to assign to the strategies of an opponent. And statisticians, in statistical decision theory, decided it was meaningless to talk about the "probability" that a ~~given~~ certain parameter had a certain value, when asa matter of fact just one value was ~~true~~ "true" and all others "false."

16. These game-theorists and statisticians devised models ~~of~~ to represent the decision-situation in which probabilities played no role: the payoff matrix, with such decision rules as minimax, minimax regret, and the Hurwicz criterion.

17. ~~Shackle, arguing along the sa~~ Such models seemed to provide an alternative to the calculated risk approach, even outside the area they were designed for. To be sure, they don't even allow for an ordering of likelihoods. But Shackle, arguing along the same lines as Knight, goes even further than Knight to say that in many ~~situatio~~ one-shot situations, people don't act as if they distinguished between likelihoods at ~~al~~ all. In these cases, he ends up with a model that is virtually equivalent to the Hurwicz model.

18. ~~So~~ But all these approaches rule out numerical probs mainly on the basis of the fact that the feelings of likelihood can't be measured on the basis of statistical frequencies. ~~They~~ They ignore any possibilities for ~~ma~~ making numerical probs meaningful ~~on in~~ by some other

type of measurement. In fact, they go to the point, after ruling out statistical experience and thence numerical probs, of maintaining that the subject takes no account of probs at all. Thus, Shackle explains what he means when he says that statistical frequenceis are irrelevant to the individual choice by an example like this:

Test Match

19. It seems obvious that we can give a better answer than that. Specifically, we will say, "England was probably bat first," or it pressed for a definite answer, "England will bet first." The point to emphasize is not the particular answer--which in this case does rely closely on statistical information--but the fact that we will distinguish between likelihoods. ~~Does Shackle really mean to advise the predict that D~~ We could give ~~behavioral meaning to~~ behavioral evidence of our belief by betting that England would bat first; in fact, the odds we would offer would give an indication of the degree of our belief. Can Shackle really believe that we will, or should, be ~~willing~~ equally willing to bet either way?

Runyon: the race is not always to the swift

20. It was Ramsey who first developed the ~~dix~~ idea that, since willingnes to bet at given odds was evidence for the strength of our beliefs, it provided a means for measuring ~~it~~ them.

"The degree of a belief is..the extent to which we are prepared to act upon it." ---just as an electric current was first measured by the deflection of a magnetic needle.

The idea is old: making a man prove his stated beliefs by acting onthem. "If you think there's no poison in the soup you drink it first." Most economists must sooner or later face the taunt, "If you're so smart, why ain't you rich?" "Put your money where your mouth is"; "Put up or shut up"; "Show the courage of your convictions"; "What odds will you give me?" As Ramsey puts it: "The probability of 1/3 is clearly related to the kind of belief which would lead to a bet of 2 to 1."

21. Of,course, the person's choices reveal not only his degrees of belief but his degrees of preference; there is the problem of distinguishing between them. But under certain conditions, if you pick the right choices to observe, this can be done.

21. For instance, suppose we can find an event such that a person will be indifferent between:

$$\begin{array}{c} \\ I \\ II \end{array} \begin{pmatrix} E & \sim E \\ 50 & 100 \\ 100 & 50 \end{pmatrix} \qquad \begin{pmatrix} Cad & \overset{Ford}{\cancel{Chev}} \\ \overset{Ford}{\cancel{Chev}} & Cad \end{pmatrix}$$

This isn't easy, but it has been done. QED-QEJ Now, using this event, ~~find an outcome x so that he is indifferent between~~ compare his preference for two alternatives like these

$$\begin{array}{c} I \\ II \end{array} \begin{pmatrix} 50 & 100 \\ 70 & 70 \end{pmatrix} \qquad \begin{array}{cc} E & \sim E \end{array} \begin{pmatrix} Cad & \overset{Ford}{\cancel{Chev}} \\ Chev & Chev \end{pmatrix}$$

If he prefers, we say that the "distance" in liking between.... is greater. If he is indifferent, the "intervals" are equal.

A AB > BC A B C D

50 B

~~No~~ Under certain conditions, we can arrive at a numerical scale for certain utilities; we can then use to derive numerical restrictions on other ~~utilit~~ probabilities.

22. So, starting from a choice-situation represented by a payoff matrix, we have inferred from his choices numerical utilities and probabilities. Where does this leave the notion of "uncertainty"?

23. Well, it has certainly ~~included pushed~~ enlarged the areas of "risk," since in many situations the uncertainties are clearly less "unmeasurable" than Knight and Shackle maintained, even though statistical knowledge may be lacking. Is there anything left? Yes: "measurability" by this approach is possible only when certain conditions are met; i.e., a person's choices must obey certain general patterns. And if he should follow ~~the game~~ suchdecision rules as minimax, minimax regret, the Hurwicz criterion or Shac kle's hypothesis, he will not meet those conditions. (or if he violates the conditions in any other manner).

24. So we do still have two theoretical approaches. The question is: when is one or the other appropriate?

If I do nothing more this afternoon than to raise these questions in your mind, I will have failed, But what I intend to present are answers.

25. Emphasis on "plausibility," reasonableness. What would a reasonable man do? This is an acceptable source of hypotheses, but not the only one. Shackle:

"The task we are charged with is not that of designing the human mind but of describing and understanding,how, overtly, it proceeds."

26. Even ~~game theorists~~ backers of game against Nature tend to restrict approach to cases where it is reasonable: when person has "no idea" what likelihoods are. But this is rare; even in statistical and game case. Even mathematicians have to bet sometimes on the truth or falsity of an unproved theorem--as when they decide whether or not to go on reading proofs of it. For instance, squaring the circle was only proved impossible in 1885, but ~~before~~ long before then the French Academy of Sciences was rejecting all treatises purporting to provide a proof/ Poincare remarked that the Academicians couldn't have proved they were right in doing so, but they would have provided some argument such as:

"We have compared the probability that an unknown scientist should have found out what has been vainly sought for so long, with the probability that there is one madman the more on the earth, and the latter has appeared to us the greater."

27. ~~The~~ I will reveal my answer at this point in all its naked implausibility; then try to clothe it in some bits of precedent, in the form of various offhand remarks by economists and some experimental work by psychologists. Formally, it solves our problems; it is also, I think, true.

28. The axiom I propose to discard is the independence of subj. probs and utilities;

Rationalist: this seems one of the most reasonable. Experimentalist: but this makes it impossible to infer probs from choices; true; nasty break. But this doesn't leave us wallowing without a model; for specific forms of interaction, the game models will serve. Minimax; Hurwicz.

Game-theorist: but this is an arbitrary interpretation; why talk about expectations at all, why bring in this unnecessary "as if" assumption?

Because I think that to take these models as our startingpoint is to lose sight of essential features; it is these models that make the "as if" assumptions, not the approach that starts with expectations. These models are useful, when they are, just because they express a dependence of expectation on payoff.

25. Consider Kaysen's approach to the game problem, starting with expectations. weights: expectations; asiom that if one outcome increases preference for that stratshould increase. Consider measuring pref by frequency of choice.

Now consider game solution:

$$\begin{pmatrix}10 & 0\\ 1 & 1\end{pmatrix}\begin{matrix}0\\ 1\end{matrix}\qquad \begin{pmatrix}10 & 0\\ 1 & 3\end{pmatrix}\begin{matrix}\frac{1}{6}\\ \frac{5}{6}\end{matrix}\qquad \begin{pmatrix}10 & 0\\ 1 & 2\end{pmatrix}\begin{matrix}\frac{1}{11}\\ \frac{10}{11}\end{matrix}\qquad \begin{pmatrix}100 & 0\\ -1 & 1\end{pmatrix}\begin{matrix}\frac{1}{51}\\ \frac{50}{51}\end{matrix}$$

$$\begin{pmatrix}0 & 0\\ -1 & 0\end{pmatrix}\begin{matrix}1\\ 0\end{matrix}$$

10. I propose to single out one axiom as crucial, and to suggest under just what conditions it may be violated. ~~It is~~ The behavior that violates it lies outside the risk models, yet it need not be random, unpredictable; in fact, the game models seem to express it well. Thus, what I think I have is a theory that specifies the empirical boundaries of the two approaches, thus encompasses them. The non-probabilistic area is then covered by the "Theory of the Uncalculated Risk."--it corresponds to Knight's "uncertainty", not merely because ~~it~~ like his it does not employ probs, but because many of the empirical considerations that led him to reject statistical probs will also operate against the measurement of subjective probs.

11. Consider games, where the non-prob model is most accepted. We have already rejected the arguments von Neumann and Morg use. How then justify the model? Consider Kaysen's modification. Plausible. Consider preference of one strat over another to be indicated by frequency of choice--psych technique. Kaysen's axioms.

12. Consider minimax mixed strats as payoffs change. Clearly they violate Kaysen's axioms. But can we see a pattern--and can we see any "plausibility" (realism) in it? We could say he abandons the strat as it is "improved"--which doesn't seem reasonable or likely. Or we could say he acted as if improving an outcome-- or a column--made it less likely.

Would this be a "reasonable" way to act? Yes--because it is true that that outcome becomes more unlikely--if the opponent is reasonable and informed. We might doubt that there is any reason for the relationship between payoffs and probs to be quite so precise, mechanical, and pessimistic--for payoffs sufficiently large it might well be worth gambling that the opponent will be uninformed or stupid or rigid--but the model does express an important aspect of games. From this point of view it is most appropriate where the opponent is known to be influenced by the payoffs and where the effect of those payoffs on his choice can be most clearly predicted; in such cases, it might well be that ~~the influence~~ the player's expectations (of his opponent's behavior) are so strongly controlled by the payoffs that--as an approximation--his expectations can be wiped out of the picture and his choice described entirely in terms of the payoffs--i.e., as though he ~~minimaxed~~ made no probability assumptions. Note that it would be impossible to infer probs from his choices.

13. But what does this tell about non-game situations? After all, the game situation is the one situation in which the "objective" prob. of your outcome varies with its size; i.e., it is the one situation where it is "reasonable" to "assume" that likelihood depends on payoff. But I haven't talked about "objective" prob at all; I have merely pointed out that the subjective prob in the minds of actual people actually does depend on payoffs in the game case. Is this the only case in which people act "as though" likelihoods depended on payoffs? Is "game opposition" essential?

14. No. ~~There are situations in~~ We can find even in ec literature the argument that people in non-game situation will and reasonably should act as if probs depended on payoffs.--in fact, in the same manner as in game situations.

Knight: "The action which follows upon an upoion depends as much upon the amount of confidence in that opinion as it does upon the favorableness of the opinion itself...Fidelity to the actual psychology of the situation requires, we must insist, recognition of these two separate exercises of judgment." This implies that there are decisions in which the "estimates of likelihood" are in some sense the same, but which differ because "confidence" differs.

Difficulty of making this precise: if prob is a form of prob, we can just compound probs; on the other hand, if probs are inferred from choices, then if choices differ, this just contradicts our assumption that the "likelihoods" were the same.

But if we have some indicator of "confidence,2 outside the choice situation, we can pick choices that differ only in this level of "confidence" and see whether behavior differs; see whether we get different estimates of subjective prob.

For instance, take Knight's example:

"One man knows that there are red and black balls in an urn, but is ignorant of the numbers of each; ~~.xxItxmustxbexadmittedxthat~~ Suppose we are allowed to look into the urn containing a large number of black and red balls before making a wager, but are not allowed to count the balls" or suppose we had an impression of equal numbers; suppose finally we counted the balls and found them equal.

These three situations differ, we might expect, in "confidence"; or "ambiguity." Will this variable ~~introduce~~ have effects on the man's behavior that will not fit the Savage theory of the calculated risk? Knight clains that for the first case, "It must be admitted ~~as a practical matter~~ that, practically, if any decision as toconduct is involved, such as a wager, the first man would have to act on the supposition that the chances are equal."

This ~~is~~ means he will be indifferent between betting on red and black. Likewise, in ~~urn~~ other urns.

The Savage theory appoies prob $\frac{1}{2}$ to the event Red for all three urns; it implies that he is indifferent between a bet on Red from the first urn to a bet on Red from the third urn (or second). IS HE?

28. Chipman has tested this. The answer is no; in fact, in his experiment they much preferred the urn with the known proportions. Thus, they violate some of the axioms. What is there in the situation which might have led us to predict this, and to predict their actual behavior?

29. Old notion in economics that where information is low, or confidence in estimate is low, or ambiguity is high, this will influence action upon the judgments of likelihood.

Fellner: "It is impossible to attach definite probs to the correctness of the judgment itself. Individual allowances must be made for the possibility of surprise, that is, for deviations from best guesses." As Fellner describes it, these allowances are made by operating on the ~~pro~~ likelihood weights attached ~~the~~ to the outcomes; and there is a strong presumption that the possibility of less favorable outcomes is weighted heavier than the contrary.

This would explain Chipman's results; person acts as though Red were less likely from the unknown urn than from the known urn, ~~ifxitxi~~ Red is favorable; but he would act as if Black is less likely than Black if Black were favorable.

30. He is not acting "as though" he assigned equal probs; he is not acting as though he assigned any probs indpendent of payoffs, He might be approximately ~~minimaxing; or by varying payoffs~~, we could measure ~~amount by which he is varying~~ the prob for given payoff or we could hypothesis that the prob is affected in a specified way by the payoffs. (he surely isn't really certain that Black will come up--if payoffs are fairly low).

31. Savage on vagueness: "There are practical circumstances in which one might well be willing to accept the rule--even one who, like myself, holds a personalistic view of probability. It is hard to state the circumstances precisely, indeed they seem vague almost of necessity 2" The cirumstances he suggests are when the possible losses are large, and when an "aura of vagueness" is attached to the judgments of probability. In thiese cases a minimaxer (regret) would justify himself by saying, "I do not feel I know my own mind well enough to act definitely" on my estimates of likelihood.

32. The reult means more than that "vague" problems are distorted in some way; the fact that they are distorted in a way that depends on the payoffs means that they can't be deduced from observation of choices.

We must take into account one more variable: importance. Suppose that the "distortion" operates more at high levels of payoffs, large possible losses, than at small. Such a man will contradict the Independence Axiom that underlies Savage's basic notionof qualitative probability.

$$\begin{pmatrix} 5 & 10 \\ 10 & 5 \end{pmatrix} \qquad \begin{pmatrix} -100 & 5 \\ 5 & -100 \end{pmatrix}$$

Luce notion:

xxAt low stakes, he may prefer to bet on some ambiguous event rather ~~33x~~ than on heads; at high stakes, if he must bet, he might prefer to bet on heads rather than on the ambiguous event or, its negative!

33. So: given vagueness and importance, even "reasonable" people will be influenced by payoffs. But always in the same direction? Let's turn to the social psychologists and their studies of actual decision-making.

34. First, note striking difference between their approach and econs: they tend to assume that expectations will be influenced by payoffs, desires, motivations. One article begins:

"It is a truism in social psychology that what one wishes determines what one believes" but starting from the opposite end of the scale, they come to the same conclusions I have suggested: "knowledge and experience also influence beliefs. Certainly all persons do not think wishfully to an equal degree, nor does any person think wishfully to the same degree on all questions."

The precise variables that influence the extent of wishful thinking--i.e., the appropriateness of non-prob models versus calculated risk models--are in each case found to be ambiguity or vagueness, and importance.

35. There are several methods for controlling or mesuring vagueness, ambiguity; it is worth mentioning that sheer quantity of information is not so important as its nature--whether or not there is conflicting information.

Thus, given amount of info can be more or less ambiguous: CIA versus Sputnik. Also, mere fact of wide difference of opinion, especiablly among neutrals, is indicative.

36. The Theory of the Uncalculated Risk then says:

1. In situations where ambiguity is high and importance is high, individuals will violate the Savage axioms: in particular, the Independence axiom.
2. The decisions based on these dependent probs and utilities may be represented by existing non-probabilistic models, such as minimax, maximax.
3. Suggestion that such situations are precisely the situations in which these models are appropriate.
4. Indications that the maximax model may really be more appropriate in actual decision-making, as opposed to SOP's; i.e., Theoryof the Uncalculatee Risk is probably close to a theory of wishful thinking.

Churchill: Facts are better than dreams.

Ego-involvement; achievement; the Crap-shooter's fallacy; capitalism and Knight's paradox.